



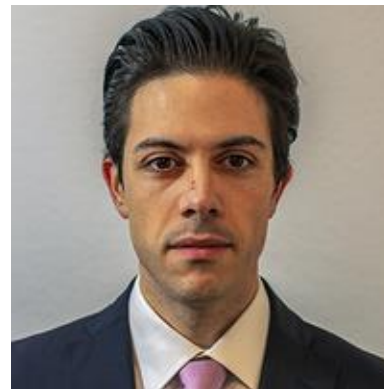
THE UNIVERSITY
of NORTH CAROLINA
at CHAPEL HILL

Quantifying the Effect of Multivalent Ions in Polyelectrolyte Solutions

Presented by Michael Jacobs



Michael Jacobs



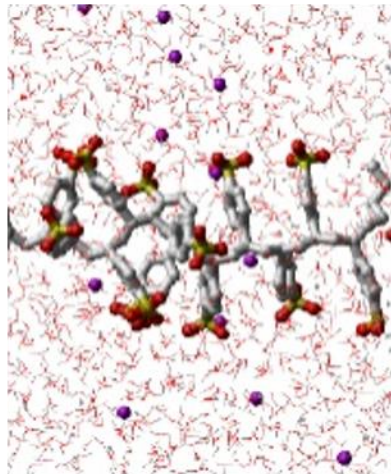
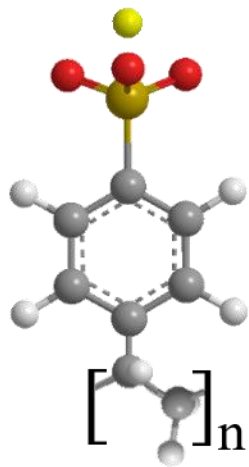
Carlos G. Lopez
RWTH Aachen University



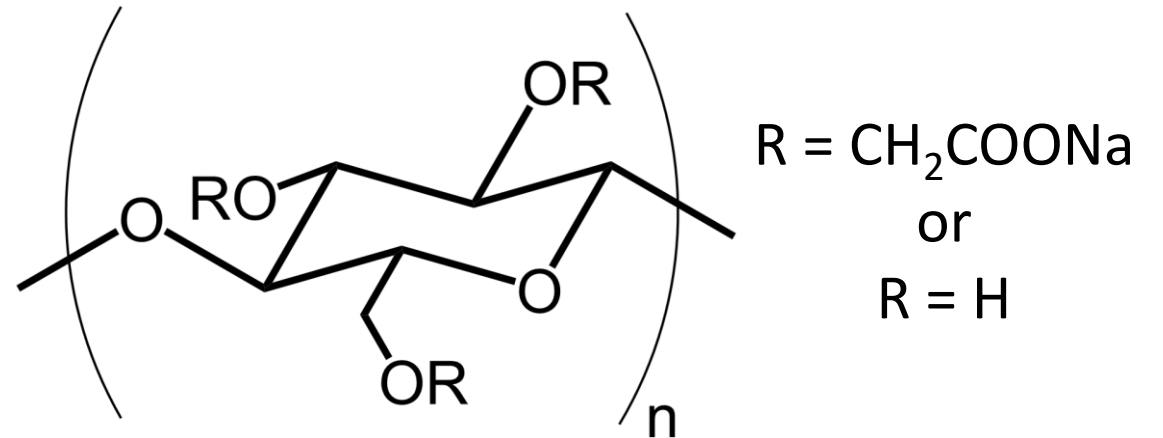
Andrey V. Dobrynin

Polyelectrolytes

Sodium Poly(styrene sulfonate)
(NaPSS)

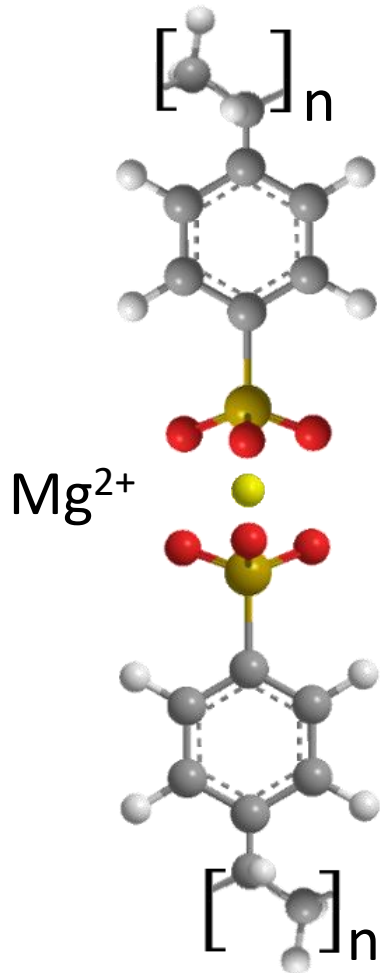


Sodium Carboxymethyl Cellulose
(NaCMC)

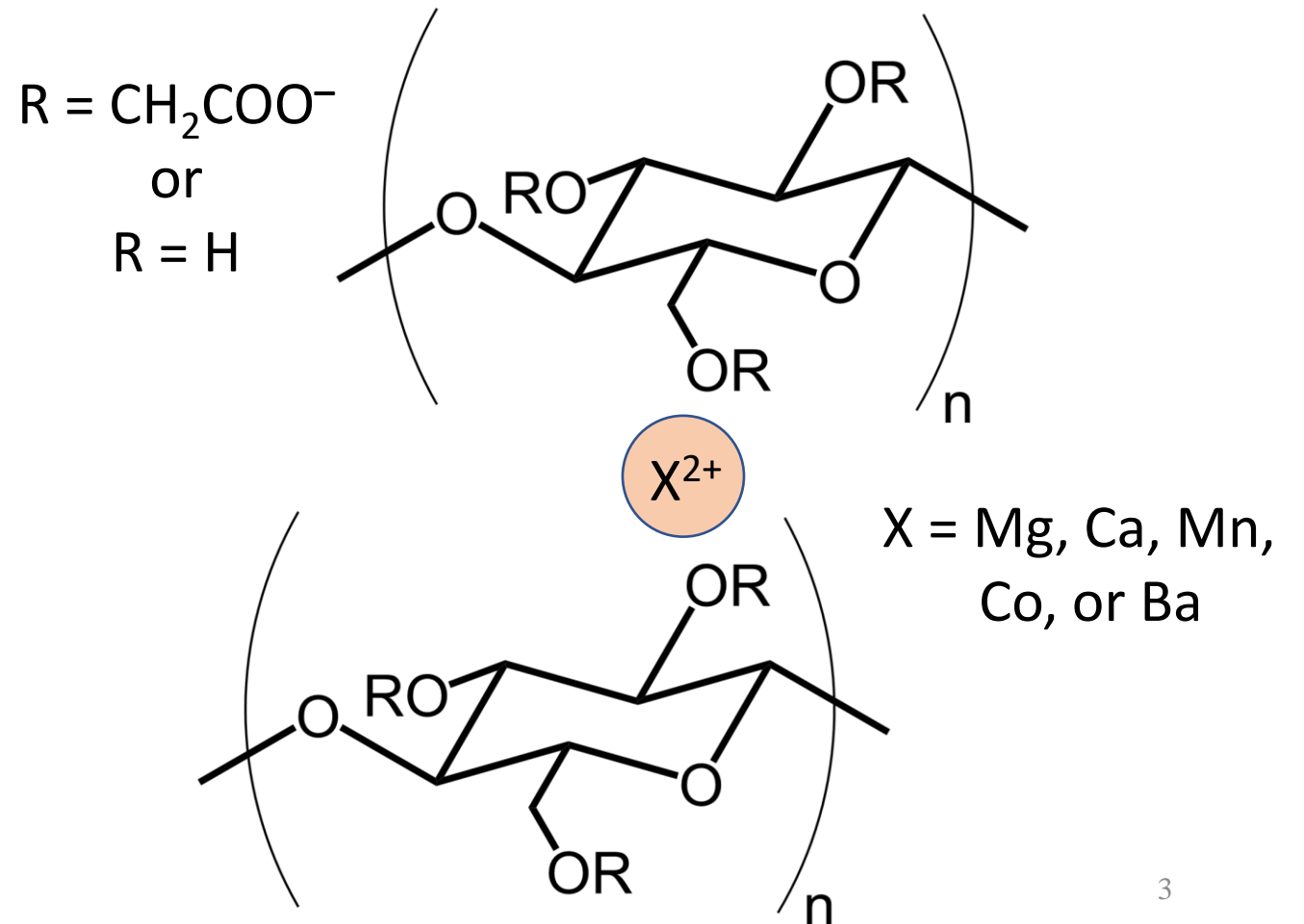


Polyelectrolytes with Divalent Counterions

Magnesium Poly(styrene sulfonate)
(MgPSS)

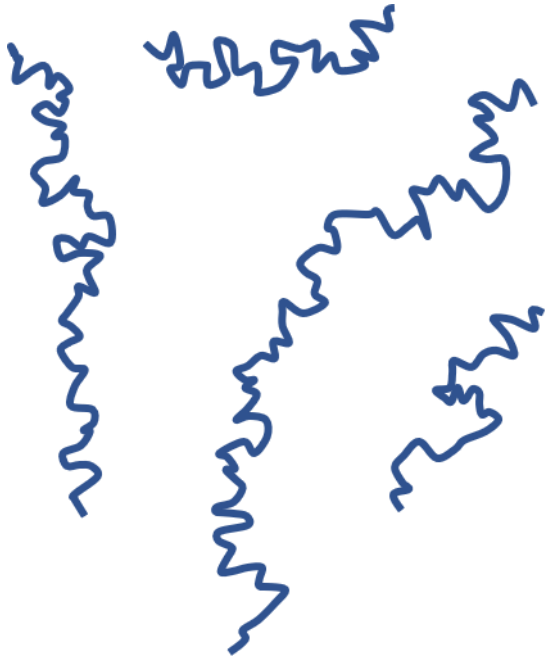


X^{2+} Carboxymethyl Cellulose
(X^{2+} CMC)



Church of the Holy Blob

Chains in Semidilute Solutions



Chain Size

$$R = \xi(N_w/g)^{0.5}$$

Correlation Length (Blob)

$$\xi = D_e g/g_e = l g/B_{pe}$$

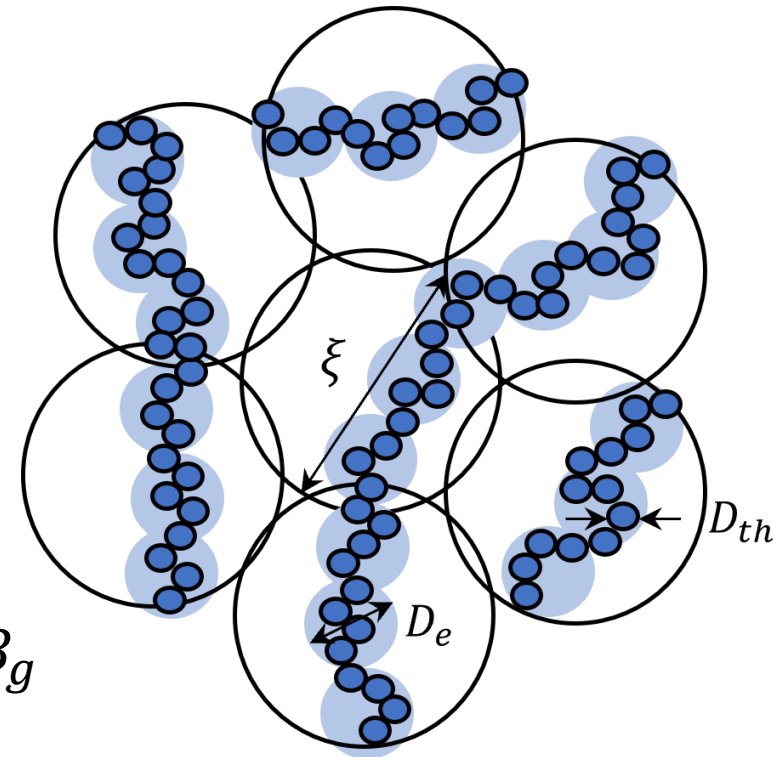
Electrostatic Blob

$$D_e = D_{th}(g_e/g_{th})^{0.588} = l g_e^{0.588}/B_g$$

Thermal Blob

$$D_{th} = (l b g_{th})^{0.5} = l g_{th}^{0.5}/B_{th}$$

Chains of Blobs



N_w – weight-average degree of polymerization

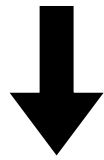
l – monomer projection length

b – Kuhn length

Correlation Length

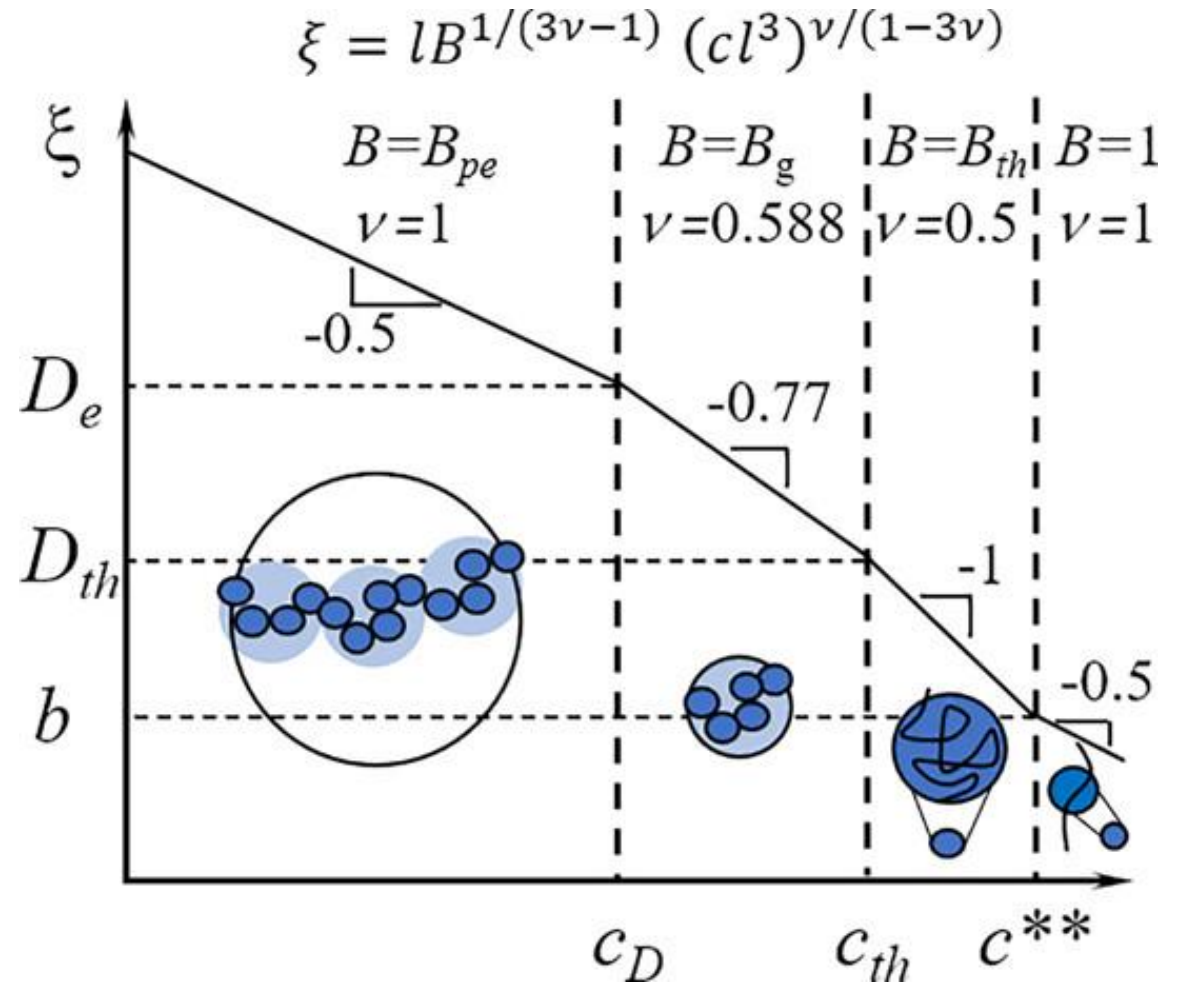
$$\xi = lg^\nu / B$$

Space-filling condition
 $g = c\xi^3$



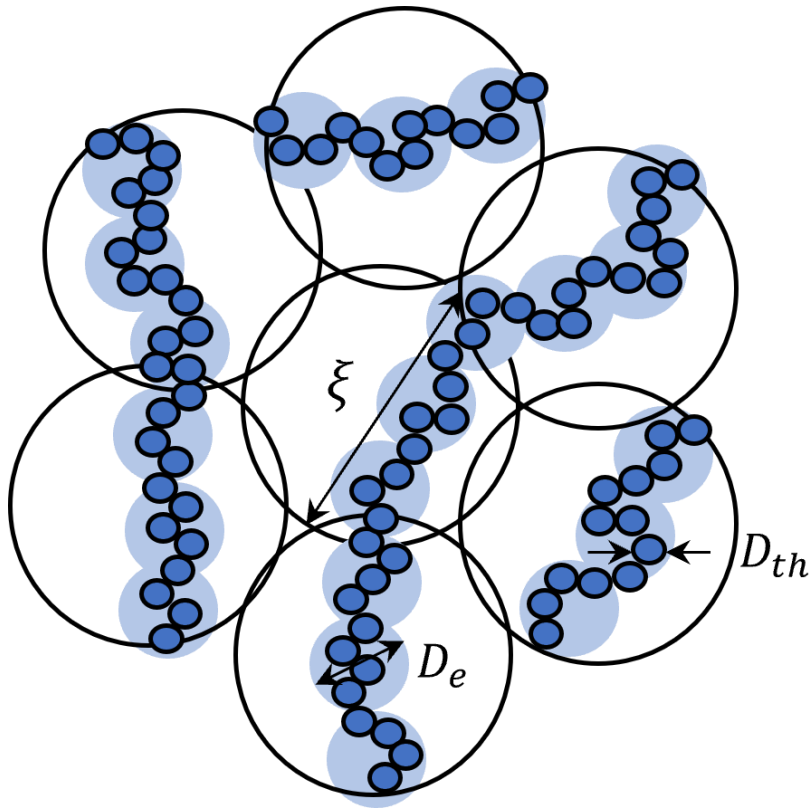
$$\xi = lB^{1/(3\nu-1)} (cl^3)^{\nu/(1-3\nu)}$$

$$g = B^{3/(3\nu-1)} (cl^3)^{1/(1-3\nu)}$$



Unentangled (Rouse) Dynamics

Chains of Blobs



Blob Relaxation Time

$$\tau_\xi = \frac{\eta_s}{k_B T} \xi^3$$

Terminal Shear Modulus

$$G = k_B T c / N_w$$

Longest Chain

Relaxation Time

$$\tau_R = \tau_\xi \frac{N_w^2}{g^2} = \frac{\eta_s}{k_B T} \frac{N_w^2}{g c}$$

Solution Viscosity

$$\eta - \eta_s = G \tau_R = \eta_s N_w / g$$

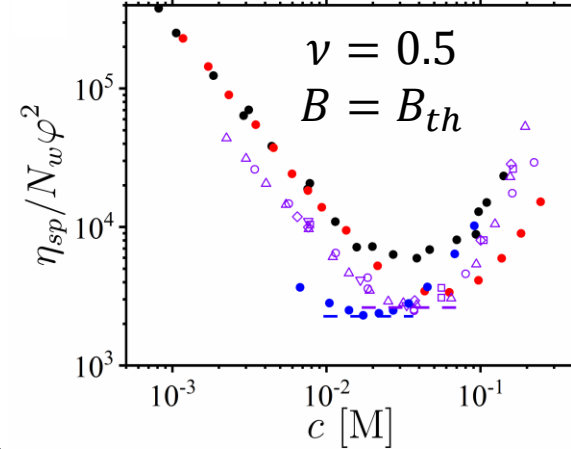
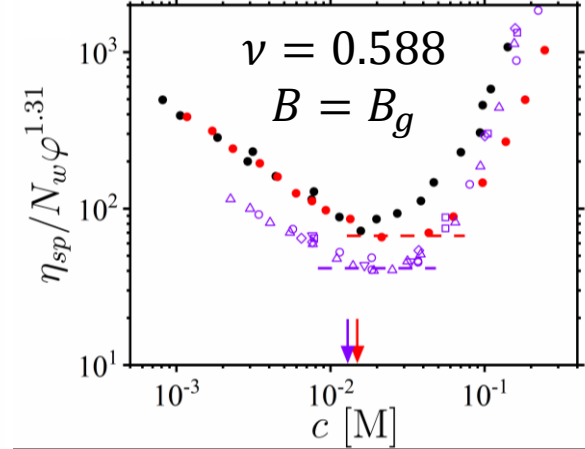
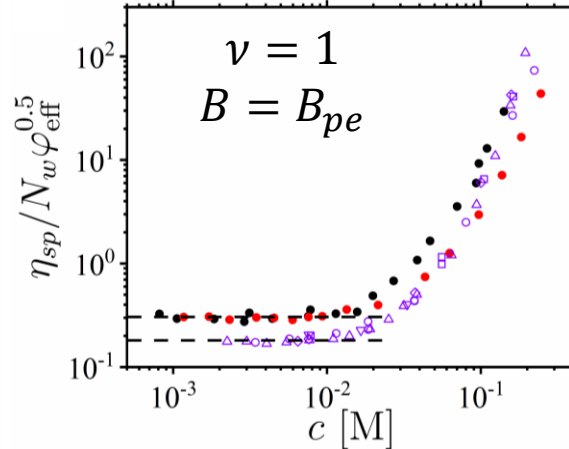
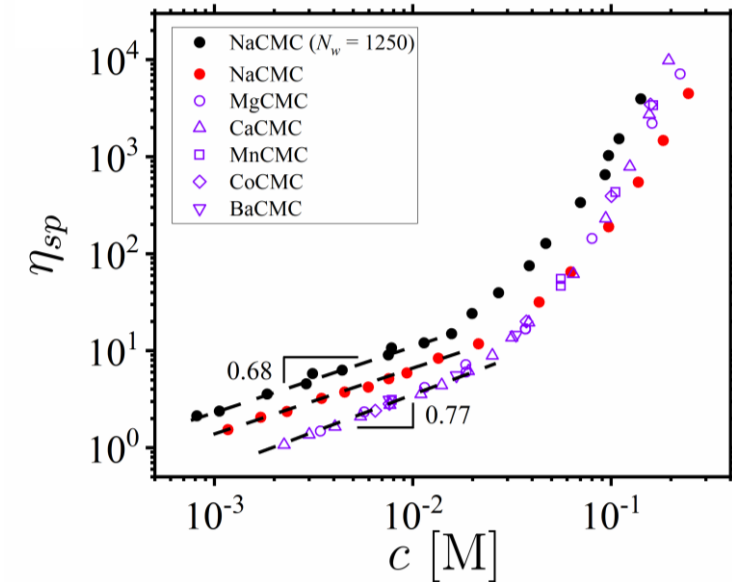
Specific Viscosity

$$\eta_{sp,R} = \frac{N_w}{g}$$

Viscosity Data Analysis

Aqueous solutions of NaCMC and X²⁺CMC with $N_w = 720$ and 1250 and $l = 0.515\text{nm}$.

$$\eta_{sp,R} = N_w/g = N_w B^{3/(1-3\nu)} (cl^3)^{1/(3\nu-1)}$$



$$\varphi_{\text{eff}}^{0.5} = \varphi^{0.5} (1 + 2c_s/zf^*c)^{-0.75}$$

Low salt correction
(*Macromolecules* **1995**, 28, 1859)

$$g = l B_{pe}^{1.5} (cl^3)^{-0.5} \left(1 + \frac{2c_s}{zf^*c} \right)^{0.75}$$

Residual salt concentration

$$c_s/f^* = 3.3 \times 10^{-4} \text{M (Na}^+)$$

$$c_s/f^* = 2.0 \times 10^{-4} \text{M (X}^{2+})$$

$$B = C_p^{1/3-\nu}$$

$$\varphi = cl^3$$

Extracting Molecular Parameters

Kuhn length

$$b = l B_{th}^{-2}$$

2.2nm (NaPSS)



2.0nm (MgPSS)

6.8nm (NaCMC)



7.1nm (X²⁺CMC)

Excluded volume

$$v = (lb)^{1.5} \left(\frac{B_g}{B_{th}} \right)^{\frac{1}{1-2\nu}}$$

0.11nm³ (NaPSS)

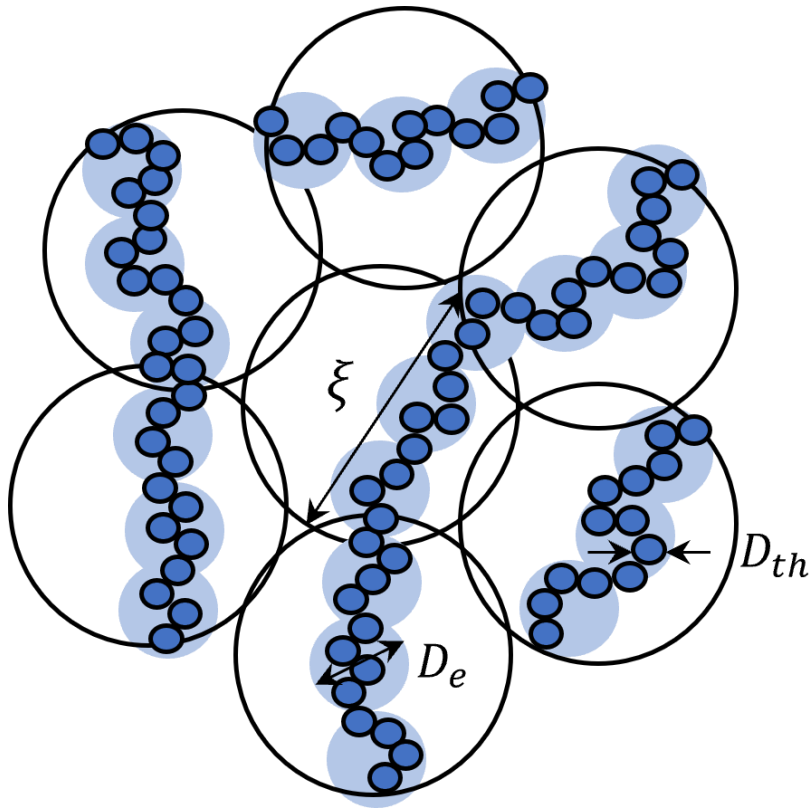


0.052nm³ (MgPSS)

1.77nm³ (NaCMC)



0.89nm³ (X²⁺CMC)

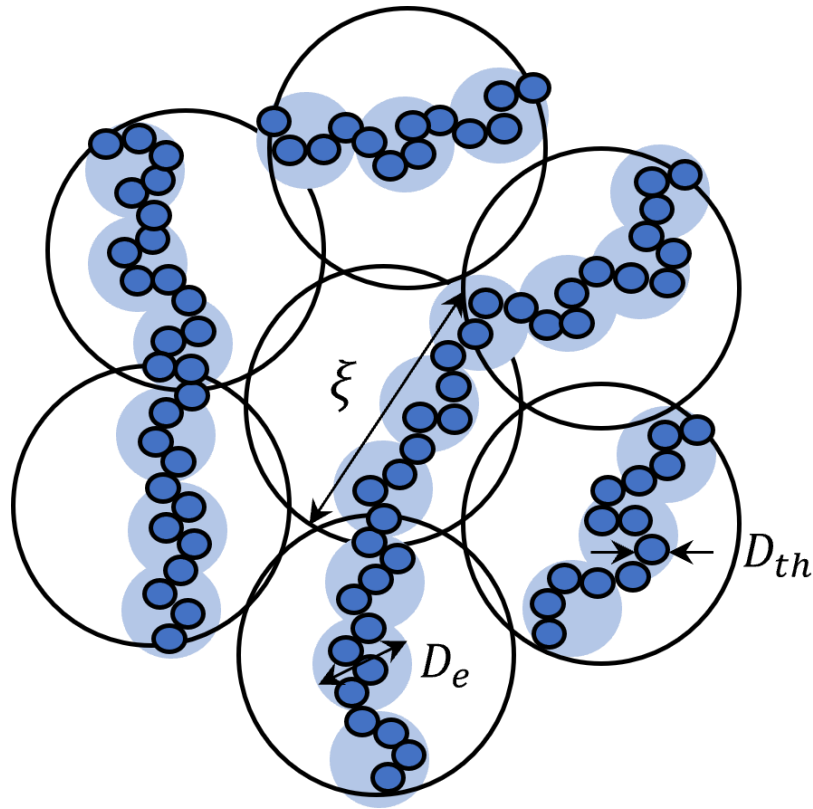


Free Counterions

Fraction f^* of free counterions:

$$\frac{l_B (f^* g_e)^2}{D_e} k_B T \approx 4 k_B T \Rightarrow f^* \approx \frac{2}{g_e} \left(\frac{D_e}{l_B} \right)^{0.5}$$

Bjerrum length in water $l_B = 0.7 \text{ nm}$



7.1% (NaPSS)



2.0% (MgPSS)

11.8% (NaCMC)



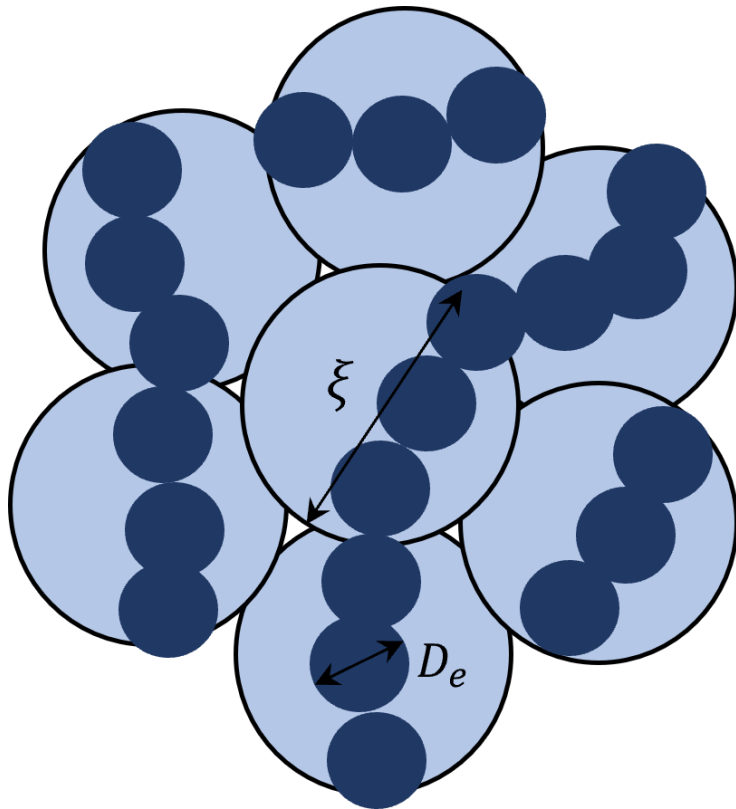
6.8% (X^{2+} CMC)

Free Counterions

Fraction f^* of free counterions:

$$\frac{l_B (f^* g_e)^2}{D_e} k_B T \approx 4 k_B T \Rightarrow f^* \approx \frac{2}{g_e} \left(\frac{D_e}{l_B} \right)^{0.5}$$

Bjerrum length in water $l_B = 0.7 \text{ nm}$



Counterions are largely condensed inside the electrostatic blobs

7.1% (NaPSS)



2.0% (MgPSS)

11.8% (NaCMC)



6.8% (X^{2+} CMC)

Conclusions

Multivalent counterions

- do not significantly affect the Kuhn length ($\sim 10\%$)
- reduce the excluded volume by a factor of two
- reduce the fraction of free counterions

Acknowledgements

Collaborators



Ryan Sayko

Quantifying Properties of Polysaccharide Solutions

9:12



Andrey V. Dobrynin



Carlos G. Lopez

RWTH Aachen University

Solution Rheology of Polyelectrolytes with Monovalent and Divalent Counterions

10:00

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